Pi equals 0, or, how I learned to stop worrying and let delta depend on x

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Lemma. $\lim_{x \to 0^+} (x) = \pi$

Proof. Suppose $\varepsilon > 0$. We must find δ such that $0 < x < \delta$ implies that $|x - \pi| < \varepsilon$. Let $\delta = -x^2 + (2\pi + 1)x + \varepsilon^2 - \pi^2$. Then:

$$\begin{aligned}
0 < x < \delta \quad &\Rightarrow \quad x < -x^2 + (2\pi + 1)x + \varepsilon^2 - \pi^2 \\
&\Rightarrow \quad 0 < -x^2 + 2\pi x + \varepsilon^2 - \pi^2, \text{ apply quadratic formula:} \\
&\Rightarrow \quad x \text{ is between } \frac{-2\pi \pm \sqrt{(2\pi)^2 - 4(-1)(\varepsilon^2 - \pi^2)}}{-2(-1)} \\
&\Rightarrow \quad x \text{ is between } \pi \pm \varepsilon \\
&\Rightarrow \quad |x - \pi| < \varepsilon
\end{aligned}$$

It follows that $\lim_{x \to 0^+} (x) = \pi$.

Theorem. Since x is a continuous function, $\lim_{x \to 0^+} (x) = 0$. Therefore, $\pi = 0$.

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